International Journal of Mathematics, Statistics and Operations Research
Vol. 4; Number 1; 2024, pp. 67-76
https://doi.org/10.47509/IJMSOR.2024.v04i01.05

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# Multi - Objective Single Machine Scheduling Problem Under Fuzzy Programming Technique

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**Abstract.** Single Machine Scheduling Problem (SMSP) is one of most important problems in manufacturing, because there are finite jobs (tasks) and limited resources (machine). Therefore, SMSP can provide good ideas for complex system. In this paper, we propose fuzzy programming technique for a Multi-Objective Single Machine Scheduling Problem (MOSMSP) when processing times of jobs are normal random variables. The probabilistic problem is converted into an equivalent deterministic programming problem. Then the fuzzy programming technique has been applied to obtain a compromise solution. A numerical example demonstrates the feasibility of applying the proposed model to single machine scheduling problem.

**KEYWORDS**
Fuzzy programming, Single machine scheduling, Processing time, Normal random variables, Probabilistic and deterministic programming problem.

## 1. Introduction

Optimization is a central concept in operations research, and it involves finding the best possible solution to a problem that satisfies given constraints. Optimization techniques are used in a wide range of fields, including engineering, finance, transportation, logistics, and manufacturing Vikas et al. [2]. Single Machine Scheduling Problem (SMSP) is one of the most important problems in manufacturing, because there are finite jobs (tasks) and limited resources (machine). Therefore, SMSP can provide good ideas for complex system. Scheduling consists of planning and arranging jobs in an orderly sequence of operations in order to meet customer's requirements.

The scheduling of jobs and the control of their flows through a production process are the most significant elements in any modern manufacturing systems. The single machine environment is basis for other types of scheduling problems. In a single machine scheduling, there is only one machine to process all jobs so that optimizes system performance measures such as make-span, completion time, tardiness, number of tardy jobs, idle times, sum of the maximum earliness and tardiness. In single

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**Article History**
Received : 24 March 2024; Revised : 28 April 2024; Accepted : 15 May 2024; Published : 29 June 2024

**To cite this paper**
Vikas S. Jadhav, Sumedh U. Buktare & Madhav R. Fegade (2024). Multi-Objective Single Machine Scheduling Problem under Fuzzy Programming Technique. *International Journal of Mathematics, Statistics and Operations Research.* 4(1), 67-76.

machine scheduling, most of the researches are concerned with the minimization of a single criterion. However, scheduling problems often involve more than one aspect and therefore, they require multiple criteria analysis Reza [1].

In fact, the decision maker often wants to minimize the weighted completion time and total weighted tardiness. Each of these objectives is valid from a general point of view. Since these objectives conflict with each other, a solution may perform well for one objective or it gives inferior results for others. For this reason, scheduling problems have a multi-objective nature. In decision making situations, the high degree of fuzziness and uncertainties is included in the data set.

## 2. Review Of Literature

Various researchers have done a lot of work in different directions. Johnson [3] has proposed a basic algorithm for n jobs, two machine scheduling problem with minimizing make-span. Brown, Lomnicki [4] introduced the concept of flow shop scheduling with the help of branch and bound method. V.S. Jadhav, Bajaj [5] focused on flow shop scheduling problem using fuzzy triangular membership function constrained optimization approach. Further the work was developed by Singh and Gupta [6] made an attempt to study the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria.

The fuzzy set theory provides a framework for handling the uncertainties of this type Zadeh [7]. Bellman and Zadeh [8] presented some applications of fuzzy theories to the various decision-making processes in a fuzzy environment. Zimmerman [9 and 10] presented a fuzzy optimization technique to a linear programming (LP) problem with single and multi objectives. The fuzzy set theory has been applied to formulate and solve problems in various areas such as artificial intelligence, image processing, robotics, pattern recognition, and the like Hannan, [11] and Yager, [12]. Different approaches to multi-objective single machine problems with fuzzy parameters have been presented in the literature during the last two decades.

Considered a single machine scheduling problem minimizing the V. S. Jadhav et al., Tavakkoli-Moghaddam et al., and Pappis [2, 13 and 14] presented a fuzzy-linguistic approach to multi-criteria sequencing problem. They considered a single machine, in which each job is characterized by fuzzy processing times. The objective is to determine the processing times of jobs and the common due as well as to sequence the jobs on the machine where penalty values are associated with due dates assigned, earliness, and tardiness. Another approach to solve a multi-criteria single machine scheduling problem is presented by V. S. Jadhav et al. and Pappis [14]. They proposed an approach using linguistic values to evaluate each criterion (e.g. very poor, poor, fair, good, and very good) and to represent its relative weights (e.g. very unimportant, unimportant, moderately important, important, and very important).

Very recently, V. S. Jadhav and O. S. Jadhav [15, 2] they highlighted on flow - shop scheduling problem to minimize total elapsed time under fuzzy approaches, and ranking of octagonal fuzzy numbers for solving fuzzy job sequencing problem using robust ranking technique. Chanas and Kasperski [16] studied the single machine scheduling problem with parameters given in the form of fuzzy numbers. It is assumed that the optimal schedule in such a problem cannot be determined precisely. In their paper, it is shown how to calculate the degrees of possible and necessary optimality of a given schedule in one of the special cases of the single machine scheduling problems. But here we find the best possible order (sequence) for the fuzzy sequencing problem excusive of changing traditional problems.

Thus, the aim of this paper is to develop a fuzzy programming approach for solving a multi-objective single machine scheduling problem when processing times of jobs are normal random variables and the constraints follow a joint probability distribution. This probabilistic model is first converted into an equivalent deterministic model, to which fuzzy programming technique is applied to solve a multi-objective single machine scheduling problem to obtain a compromise solution.

## 3. Mathematical Formulation of MOSMSP

The following notations and definitions are used to describe the multi-objective single machine scheduling problem.

### *3.1. Indices and Parameters*

$N =$ number of jobs,
$p_i =$ processing time of job i (normal random variable) $(i = 1, 2, \ldots, N)$,
$R_i =$ release time of job $i$ $(i = 1, 2, \ldots, N)$,
$d_i =$ due date of job $i$ $(i = 1, 2, \ldots, N)$,
$W_i =$ importance factor (or weight) related to job $i$ $(i = 1, 2, \ldots, N)$,
$M =$ a large positive integer value.

### *3.2. Decision Variables*

$$X_{ij} = \begin{cases} 1 & \text{if job } j \text{ scheduled after job } i \\ 0 & \text{Otherwise} \end{cases}$$

### *3.3. Mathematical Model*

The mathematical model of fuzzy programming formulation of single machine scheduling problem for multi-objectives and a set of constraints can be written as follows:

In this model, the objective is to find the best (or optimal) schedule minimizing the weighted completion time ( $Z_1$) and total weighted tardiness ($Z_2$) of a manufacturing system.

$$\text{Min } Z_1 = \sum_{i=1}^{N} W_i C_i \tag{3.1}$$

$$\text{Min } Z_2 = \sum_{i=1}^{N} W_i T_i \tag{3.2}$$

subject to

$$C_i \geq R_i + P_i \quad \forall i \tag{3.3}$$

$$X_{ij} + X_{ji} = 1 \quad \forall\, i, j,\ \imath \neq j \tag{3.4}$$

$$C_i - C_j + MX_{ij} \geq P_i \tag{3.5}$$

$$T_i = \text{Max}\{0, C_i - D_i\} \tag{3.6}$$

$$X_{ij} \in \{0, 1\} \tag{3.7}$$

Constraint (3.3) ensures that the completion time of a job is greater than its release time plus processing time. Constraint (3.4) specifies the order relation between two jobs scheduled. Constraint (3.5) stipulates relative completion times of any two jobs. M should be large enough for constraint (3.5). Constraint (3.6) specifies the tardiness of each job.

## 4. Multi - Objective Chance Constrained Programming Problem

The chance constrained programming was first developed by Charnes and Cooper [17]. Subsequently, some researchers like Contini [18], Leclercq [19], Teghem et al. [20] and many others have established some theoretical results in the field of stochastic programming. Sinha and Biswal et al. [21] have presented fuzzy programming problem approach to multi-objective stochastic programming problems when bi's follows joint normal distribution.

A multi-objective chance constrained programming problem with a joint probability constraint can be stated as:

$$\text{Min } Z^k(x) = \sum_{j=1}^{n} C_j^{(k)} x_j, \quad k = 1, 2, \ldots, K \tag{4.1}$$

Subject to

$$\Pr\left(\sum_{j=1}^{n} a_{1j}x_j \geq b_1,\ \sum_{j=1}^{n} a_{2j}x_j \geq b_2, \ldots \sum_{j=1}^{n} a_{nj}x_j \geq b_m\right) \geq 1 - \alpha \tag{4.2}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{4.3}$$

Where $b_i$'s are independent normal random variables with known means and variances. Equation 4.2 is a joint probabilistic constraint and $0 \leq \alpha \leq 1$ is a specified probability. We assume that the decision variables $X_j$'s are deterministic. Let the mean and standard deviation of the normal independent random variable $b_i$ be given by $E(b_i)$ and $\sigma(b_i)$, respectively. Hence the equivalent deterministic model of probabilistic problem can be presented as [21].

$$\text{Min } Z^k(x) = \sum_{j=1}^{n} C_j^{(k)} x_j, \quad k = 1, 2, \ldots, K \tag{4.4}$$

Subject to

$$\frac{3\beta_i}{3-\beta_i^2} e^{-\frac{\beta_i^2}{2}} \geq \sqrt{\frac{\pi}{2}}(2\phi(\beta_i)+1), \quad i = 1, 2, \ldots, m \tag{4.5}$$

$$\text{where,} \quad \beta_i = \frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sigma(b_i)}$$

$$\prod_{j=1}^{n} \phi(\beta_i) \geq 1-\alpha, \quad i = 1, 2, \ldots, m \tag{4.6}$$

$$\sum_{j=1}^{n} a_{ij}x_j - \beta_i \sigma(b_i) = E(b_i), \quad i = 1, 2, \ldots, m \tag{4.7}$$

$$0 \leq \phi(\beta_i) \leq 1, \quad i = 1, 2, \ldots, m \tag{4.8}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{4.9}$$

We now present the methodology to solve a multi-objective stochastic programming problem using fuzzy programming approach. The algorithm includes the following steps:

### *4.1. Proposed Algorithm*

**Step** 1: First, convert the given stochastic programming problem into an equivalent deterministic programming problem by chance constrained programming technique as discussed.
**Step** 2: Solve the multi-objective deterministic problem obtained from Step 1, using only one objective at a time and ignoring the others. Repeat the process $K$ times for the $K$ different objective functions. Let $X^{(1)}; X^{(2)}; X^{(3)}; \cdots; X^{(K)}$ be the respective ideal solutions of the $K$ objective functions.
**Step** 3: Using the solutions obtained in Step 2, find the corresponding value of all the objective functions at each of the solutions.
**Step** 4: From Step 3, obtain the upper and lower bounds ($U_k and L_k, k = 1, \cdots, k$) for each of the objective functions.
**Step** 5:Using a linear membership function, formulate a crisp model, by introducing an augmented variable formulate single objective non-linear programming problem.

Hence, the model can be formulated as:

$$\text{Max } \lambda, \tag{4.10}$$

subject to,

$$Z^{(k)}(x) + (U_k - L_k)\lambda \leq U_k, \quad k = 1, 2, \ldots, K \tag{4.11}$$

$$\frac{3\beta_i}{3-\beta_i^2} e^{-\frac{\beta_i^2}{2}} \geq \sqrt{\frac{\pi}{2}}(2y_i + 1), \quad i = 1, 2, \ldots, m, \quad \textbf{where, } y_i = \phi(\beta_i) \tag{4.12}$$

$$\prod_{i=1}^{m} y_i \geq 1 - \alpha \tag{4.13}$$

$$\sum_{j=1}^{n} a_{ij} x_j - \beta_i \sigma(b_i) = E(b_i), \quad i = 1, 2, \ldots, m \tag{4.14}$$

$$0 \leq y_i \leq 1, \quad i = 1, 2, \ldots, m \tag{4.15}$$

$$x_1, x_2, \ldots, x_n \geq 0 \tag{4.16}$$

$\lambda \geq 0$ & $\beta_1, \beta_2, \ldots, \beta_m$ are unrestricted in sign

## 5. Numerical Example

Table 1 summarizes the data that form the numerical example. We consider the following assumptions:

(1) The processing times ($P_i$) is integers and is generated from a normal distribution.
(2) The due dates ($d_i$) are computed by $d_i = \mu_{P_i} \times N \times (1 - M)$ as given in [1]. $N$ is the number of jobs and $M$ the uniformly random number between 0 and 1.
(3) The ready times ($R_i$) are generated from a uniform distribution on $[1, 10]$,
(4) The jobs' weights ($w_i$) are uniformly generated from discrete uniform distribution on $[1, 10]$.

Table 1.: Generated Data

| Jobs | $\mu_{P_i}$ | $\sigma_{P_i}$ | $d_i$ | $R_i$ | $w_i$ |
|---|---|---|---|---|---|
| $J_1$ | 5 | 2 | 13 | 7 | 3 |
| $J_2$ | 7 | 3 | 18 | 5 | 5 |
| $J_3$ | 8 | 5 | 20 | 4 | 4 |
| $J_4$ | 3 | 6 | 8 | 4 | 6 |
| $J_5$ | 2 | 4 | 5 | 9 | 4 |

is presented as follows:

$$\text{Min } Z_1 = \sum_{i=1}^{N} w_i C_i, \tag{5.1}$$

$$\text{Min} Z_2 = \sum_{i=1}^{N} w_i T_i, \tag{5.2}$$

Subject to

$$\Pr\begin{pmatrix} C_i - R_i \geq P_i, \\ C_i - C_j + MX_{ij} \geq P_i \end{pmatrix} \geq 0.85 \quad \forall i,j; i \neq j \tag{5.3}$$

$$X_{ij} + X_{ji} = 1 \quad \forall i,j; i \neq j \tag{5.4}$$

$$T_i = \max\{0, C_i - D_i\}, \quad \forall i \tag{5.5}$$

$$X_{ij} \in \{0,1\} \quad \forall i,j;\ i \neq j \tag{5.6}$$

The equivalent deterministic programming problem can be obtained for the above multi-objective stochastic programming problem by using equations. (4.4) - (4.9).

$$\text{Min } Z_1 = \sum_{i=1}^{N} w_i C_i, \tag{5.7}$$

$$\text{Min } Z_2 = \sum_{i=1}^{N} w_i T_i \tag{5.8}$$

Subject to

$$C_i - R_i - \sigma_{P_i}\beta_i = \mu_{P_i} \quad \forall i,j;\ i \neq j \tag{5.9}$$

$$C_i - C_j + MX_{ij} - \sigma_{P_i}\beta_i = \mu_{P_i} \forall i,j;\ i \neq j \tag{5.10}$$

$$1.2533141(1+2y_i)(3-\beta_i^2) \leq 3\beta_i \exp\left(\frac{\beta_i^2}{2}\right) \quad \forall i \tag{5.11}$$

$$\prod_{i=1}^{6} y_i \geq 0.85 \tag{5.12}$$

$$X_{ij} + X_{ji} = 1 \quad \forall i,j;\ i \neq j \tag{5.13}$$

$$T_i = \max\{0, C_i - D_i\}, \quad \forall i \tag{5.14}$$

$$X_{ij} \in \{0,1\}, y_i \geq 0 \quad \forall i,j;\ i \neq j \tag{5.15}$$

All the computational experiments are carried out with a branch-and-bound (B&B) method in the Lingo 8.0 software by an Intel® 1.61 GHz processor with 512 Mb RAM.

Solving the problem for objective $Z_1$ and $Z_2$ , The ideal solutions are as follows:

$$Z_1 = 422.063, \quad Z_2 = 158.843$$
$$\beta_1 = \begin{pmatrix} 1.6295 \\ 1.6295 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 1.6295 \\ 1.6295 \end{pmatrix}, \quad \beta_3 = \begin{pmatrix} 1.6295 \\ 1.6295 \end{pmatrix}$$
$$\beta_4 = \begin{pmatrix} 1.6161 \\ 1.6161 \end{pmatrix}, \quad \beta_5 = \begin{pmatrix} 1.6295 \\ 1.6295 \end{pmatrix}$$
$$y_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad y_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad y_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$y_4 = \begin{pmatrix} 0.85 \\ 0.85 \end{pmatrix}, \quad y_5 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Using the linear membership function, the following fuzzy programming problem is formulated:

$$\mu(Z_1) = \begin{cases} 1 & Z_1 \leq 422 \\ \frac{1222 - Z_1}{1222 - 422} & 422 \leq Z_1 \leq 1222 \\ 0 & Z_1 \geq 1222 \end{cases} \tag{5.16}$$

$$\mu(Z_2) = \begin{cases} 1 & Z_2 \leq 158 \\ \frac{958 - Z_2}{958 - 158} & 158 \leq Z_2 \leq 958 \\ 0 & Z_2 \geq 958 \end{cases} \tag{5.17}$$

$$\text{Max } \lambda \tag{5.18}$$

Subject to

$$1222 - \sum_{i=1}^{N} w_i G_i \geq \lambda(1222 - 422) \tag{5.19}$$

$$958 - \sum_{i=1}^{N} w_i T_i \geq \lambda(958 - 158) \tag{5.20}$$

$$C_i - R_i - \sigma_{P_i}\beta_i = \mu_{P_i} \quad \forall i, j;\ i \neq j \tag{5.21}$$

$$C_i - C_j + MX_{ij} - \sigma_{P_i}\beta_i = \mu_{P_i} \quad \forall i, j;\ i \neq j \tag{5.22}$$

$$1.2533141(1 + 2y_i)(3 - \beta_i^2) \leq 3\beta_i \exp\left(\frac{\beta_i^2}{2}\right), \quad \forall i \tag{5.23}$$

$$\prod_{i=1}^{6} y_i \geq 0.85 \tag{5.24}$$

$$X_{ij} + X_{ji} = 1 \quad \forall i, j;\ i \neq j \tag{5.25}$$

$$T_i = \max\{0, C_i - D_i\}, \quad \forall i \tag{5.26}$$

$$X_{ij} \in \{0, 1\} \quad \forall i, j;\ i \neq j \tag{5.27}$$

Solving the above fuzzy programming problem, the compromise solution can be obtained as:

$$\begin{aligned}
&\lambda = 0.59\\
&Z_1 = 749.2585 \quad Z_2 = 480.4814\\
&\beta_1 = 1.6295 \quad \beta_2 = 1.6295, \quad \beta_3 = 1.6295\\
&\beta_4 = 1.6161, \quad \beta_5 = 1.6295\\
&y_1 = 1,\ y_2 = 1\ y_3 = 1,\ y_4 = 0.84,\ y_5 = 1
\end{aligned}$$

optimal (best) sequence of jobs is shown as follows:

$$J_1 - J_4 - J_5 - J_2 - J_3$$

## 6. Conclusion

The present research paper proposes the fuzzy programming model for multi-objective single machine scheduling problem with two objectives. These two objectives are to minimize the total weighted completion time ($Z_1$) and total weighted tardiness ($Z_2$) simultaneously with joint constraints, where only processing time of jobs are considered as independent normal random variables. Using the stated procedures a probabilistic multi-objective single machine scheduling problem with joint constraints can be easily transformed into a deterministic multi-objective non-linear programming problem and then solved by the fuzzy programming technique to obtain the compromise solution.

Due to the real-world situation and satisfaction of the decision maker for the above objectives, the proposed model is solved by using LINGO software to get optimal schedule. The associated computational results have been reported to show the effectiveness of the proposed approach.

## Acknowledgment(s)

The authors are very grateful to the editor and reviewer for their comments and suggestions which are helpful in improving the research paper.

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